$$F\Delta t = mv_f - mv_0$$

MOMENTUM

Physics

Unit 4

Physics Unit 4

NAD 2023 Standard F3 (Momentum)

CREDITS

- This Slideshow was developed to accompany the textbook
  - *OpenStax High School Physics*
    - *Available for free at https://openstax.org/details/books/physics*
    - *By Paul Peter Urone and Roger Hinrichs*
    - *2020 edition*
- Some examples and diagrams are taken from the *OpenStax College Physics, Physics,* and *Cutnell & Johnson Physics* 6th ed.

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$$F\Delta t = mv_f - mv_0$$

04-01 IMPULSE AND MOMENTUM

In this lesson you will…

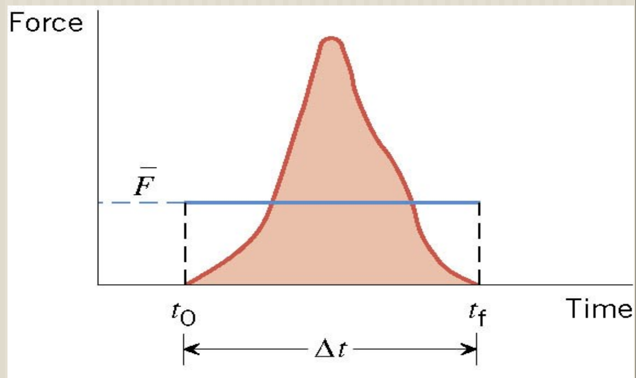
- Define impulse and linear momentum.
- Explain the relationship between momentum and force.
- Describe effects of impulses in everyday life.

OpenStax High School Physics 8.1
OpenStax College Physics 2e 8.1-8.2

04-01 IMPULSE AND MOMENTUM

$F\Delta t = mv_f - mv_0$

- Often the force acting on an object is not constant.
  - Baseball or Tennis ball being hit
- Times of force often short
- Force can be huge



04-01 IMPULSE AND MOMENTUM

$F\Delta t = mv_f - mv_0$

- To hit a ball well
  - Both size of force and time of contact are important
  - Bring both these together in concept of impulse

04-01 IMPULSE AND MOMENTUM

$F\Delta t = mv_f - mv_0$

- Impulse

$$J = F\Delta t$$

- Unit: Ns
- Is a vector

04-01 IMPULSE AND MOMENTUM

$$F\Delta t = mv_f - mv_0$$

- Object responds to amount of impulse
- Large impulse → Large response → higher v_f
- Large mass → less velocity
- Both mass and velocity play role in how responds to impulse

04-01 IMPULSE AND MOMENTUM

$F\Delta t = mv_f - mv_0$

- Linear Momentum

$$p = mv$$

- Unit: kg m/s
- Is a vector
- Is important when talking about collisions

04-01 IMPULSE AND MOMENTUM

$F\Delta t = mv_f - mv_0$

$$F = ma$$

$$a = \frac{v_f - v_0}{\Delta t}$$

$$F = m\frac{v_f - v_0}{\Delta t} = \frac{mv_f - mv_0}{\Delta t}$$

$$F\Delta t = mv_f - mv_0$$

04-01 IMPULSE AND MOMENTUM

$F\Delta t = mv_f - mv_0$

- Impulse = Change in Momentum
- Hard to measure force during contact
- Find change in momentum
  - Use impulse-Momentum Theorem and time of contact to find average force of contact

04-01 IMPULSE AND MOMENTUM

$$F\Delta t = mv_f - mv_0$$

- In the following video
  - Watch for the front of the car crumpling
  - This increase the time of the collision
  - Which decreases the force of the collision
    - Given that the change in momentum is constant

  - What other crash safety feature increases the time of collision?

$$F\Delta t = p_f - p_0$$

If the change in momentum is the same for several crashes, then the Force and time are inversely proportional.

The car crumple zone increases the time of the collision which decreases the average force of the collision.
The air bags do the same thing.

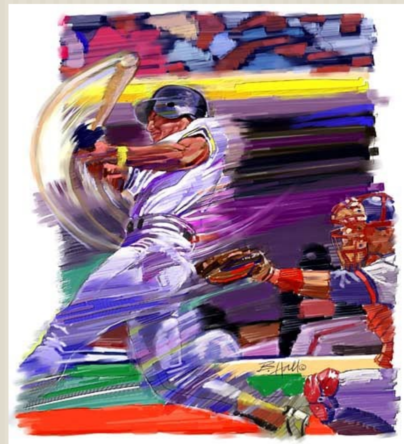
(2025 Toyota Camry updated moderate overlap IIHS crash test IIHS)

04-01 IMPULSE AND MOMENTUM

$F\Delta t = mv_f - mv_0$

- A baseball (m = 0.14 kg) with initial velocity of −40 m/s (90 mph) is hit. It leaves the bat with a velocity of 60 m/s after 0.001 s. What is the impulse and average net force applied to the ball by the bat?

- Impulse = 14 Ns
- F = 14000 N



$$F\Delta t = mv_f - mv_0$$

$$F\Delta t = (0.14\ kg)\left(60\frac{m}{s}\right) - (0.14kg)\left(-40\frac{m}{s}\right) = 14\ kg\ m/s$$

$$F = \frac{14\ kg\frac{m}{s}}{0.001\ s} = 14000\ N$$

04-01 IMPULSE AND MOMENTUM

$F\Delta t = mv_f - mv_0$

- A raindrop (m = 0.001 kg) hits a roof of a car at −15 m/s. After it hits, it spatters so the effective final velocity is 0. The time of impact is 0.01 s. What is the average force?
  - F = 1.5 N
- What if it is ice so that it bounces off at 10 m/s?
  - F = 2.5 N

$$F\Delta t = mv_f - mv_0$$
$$F(0.01\ s) = (0.001\ kg)0 - (0.001\ kg)\left(-15\frac{m}{s}\right)$$
$$F(0.01\ s) = 0.015\ kg\frac{m}{s}$$
$$F = 1.5\ N$$
$$F(0.01\ s) = (0.001\ kg)\left(10\frac{m}{s}\right) - (0.001\ kg)\left(-15\frac{m}{s}\right)$$
$$F(0.01\ s) = 0.025\ kg\frac{m}{s}$$
$$F = 2.5\ N$$

Hailstones are usually more massive than raindrops so that the force is even greater.
The rebounding adds force to the collision

04-01 IMPULSE AND MOMENTUM

$F\Delta t = mv_f - mv_0$

- In the following video
  - Old cars don't have crumple zones
  - Notice that the new car is much safer than the old car.

$$F\Delta t = p_f - p_0$$

If the change in momentum is the same for several crashes, then the Force and time are inversely proportional.

The old car doesn't have a crumple zone and the cockpit is crushed. The new car has a crumple zone and the driver is safe.

(1959 Chevrolet Bel Air vs. 2009 Chevrolet Malibu IIHS crash test)

$$F\Delta t = mv_f - mv_0$$

04-02 CONSERVATION OF MOMENTUM

In this lesson you will…

- Describe the principle of conservation of momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.

OpenStax High School Physics 8.2
OpenStax College Physics 2e 8.3

04-01B BUMPER TESTING LAB

$F\Delta t = mv_f - mv_0$

1. Each team makes a bumper out of paper and tape.
   a. 2.5 cm × 4 cm × 10 cm
   b. Do not use excessive tape
2. The bumper is placed against the end of the track.
3. The cart is released from a distance as set by the teacher.
4. The maximum force is read from the sensor.

04-02 CONSERVATION OF MOMENTUM

$F\Delta t = mv_f - mv_0$

- Do the lab in your worksheet.
- Explain why if a person standing of frictionless ice shoots a bullet at 200 m/s does not move backwards are 200 m/s.
- A 100 kg person pushes off from a 50 kg person on frictionless ice. If the 100 kg person moves at 3 m/s, what speed will the 50 kg person move at?

The person has more mass than the bullet.
Since they are half the mass, they will move at twice the speed. Mass and speed ratios are reciprocals.

04-02 CONSERVATION OF MOMENTUM

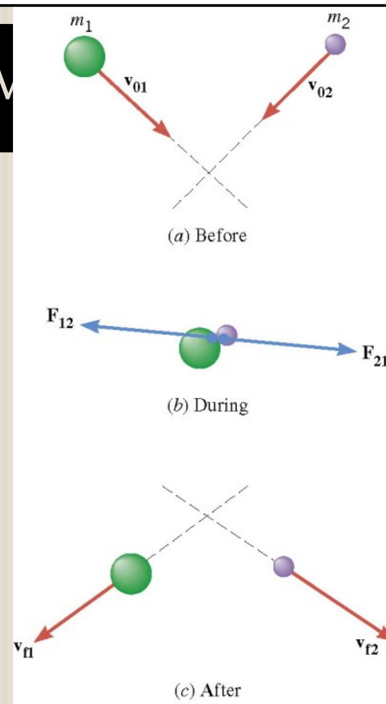
$F\Delta t = mv_f - mv_0$

- System
  - All the objects involved in the problem
  - Usually only two objects
- Internal Forces – Forces that the objects exert on each other
- External Forces – Forces exerted by things outside of the system

Usually only two objects for linear momentum because very rarely do more than two object hit at the same time. It usually happens that two hit, and then one of those hits another.
Internal forces → the objects pushing each other
External forces → gravity pulling the objects down

04-02 CONSERVATION OF MOMENTUM

- Two balls hit in the air
- During the collision
  - Internal Forces = F_{12} and F_{21}
  - External Forces = Weight (W_1 and W_2)



F_{12} and F_{21} are action-reaction pair from Newton's Third Law are equal and opposite

04-02 CONSERVATION OF MOMENTUM

$F\Delta t = mv_f - mv_0$

- $F\Delta t = mv_f - mv_0$
- Object 1: $(W_1 + F_{12})\Delta t = m_1 v_{f1} - m_1 v_{01}$
- Object 2: $(W_2 + F_{21})\Delta t = m_2 v_{f2} - m_2 v_{02}$
- Add
- $(W_1 + W_2 + F_{12} + F_{21})\Delta t = (m_1 v_{f1} + m_2 v_{f2}) - (m_1 v_{01} + m_2 v_{02})$
- $(Ext\ F + Int\ F)\Delta t = p_f - p_0$

04-02 CONSERVATION OF MOMENTUM

$F\Delta t = mv_f - mv_0$

- Since F_{12} and F_{21} are equal and opposite
  - Sum of internal forces = 0
- (External Forces)$\Delta t = p_f - p_0$
- If Isolated system:
- $0 = p_f - p_0$ OR $p_0 = p_f$

Isolated system = no external forces

04-02 CONSERVATION OF MOMENTUM

$F\Delta t = mv_f - mv_0$

- Law of Conservation of Momentum
  - In an isolated system the total momentum remains constant

$$p_0 = p_f$$

  - System can contain any number of objects

- Watch Crash Video

- Notice how the lighter car always bounces backwards.
- Conservation of Momentum predicts this

Safety consequences of vehicle size and weight by IIHS

04-02 CONSERVATION OF MOMENTUM

$F\Delta t = mv_f - mv_0$

Reasoning Strategy

1. Decide on the system
2. Identify internal and external forces
3. Is the system isolated? If no, then can't use conservation of momentum
4. Set the total initial momentum of the isolated system equal to the total final momentum

04-02 CONSERVATION OF MOMENTUM

$F\Delta t = mv_f - mv_0$

- Two billiard balls are colliding on a table. In order to apply the law of conservation of momentum, what should the system be? One ball or both billiard balls?
  - Two billiard balls.
- External Forces: Weight and Normal Force
  - If the table is horizontal these cancel
- If it were one ball, then the force of the second ball hitting it would not cancel with anything.

04-02 CONSERVATION OF MOMENTUM

$F\Delta t = mv_f - mv_0$

- A hockey puck of mass 0.17 kg and velocity 5 m/s is caught by a 0.5 kg mitten laying on the ice. What is the combined velocity after the puck is in the mitten? (ignore friction)

- v = 1.27 m/s

$$p_0 = p_f$$
$$m_1 v_{01} + m_2 v_{02} = m_1 v_{f1} + m_2 v_{f2}$$
$$(0.17\ kg)\left(5\ \frac{m}{s}\right) + (0.5\ kg)\left(0\frac{m}{s}\right) = (0.17\ kg)v + (0.5\ kg)v$$
$$0.85\ kg\frac{m}{s} = (0.67\ kg)v$$
$$v = 1.27\ m/s$$

04-02 CONSERVATION OF MOMENTUM

$F\Delta t = mv_f - mv_0$

- A 5 kg baseball pitching machine is placed on some frictionless ice. It shoots a 0.15 kg baseball horizontally at 35 m/s. How fast is the pitching machine moving after it shoots the ball?
- −1.05 m/s
- This is why you feel recoil when you shoot a gun

$$p_0 = p_f$$
$$m_1 v_{01} + m_2 v_{02} = m_1 v_{f1} + m_2 v_{f2}$$
$$(5\ kg)(0) + (0.15\ kg)(0) = (5\ kg)v + (0.15 kg)(35\ m/s)$$
$$0 = (5\ kg)v + 5.25\ kg\ m/s$$
$$-(5\ kg)v = 5.25\ kg\ m/s$$
$$v = -1.05\ m/s$$

$$F\Delta t = mv_f - mv_0$$

04-03 ELASTIC AND INELASTIC COLLISIONS

In this lesson you will…

- Describe an elastic collision of two objects in one dimension.
- Determine the final velocities in an elastic collision given masses and initial velocities.
- Define inelastic collision.
- Explain perfectly inelastic collision.

OpenStax High School Physics 8.3
OpenStax College Physics 2e 8.4-8.5

04-03 ELASTIC AND INELASTIC COLLISIONS

$F\Delta t = mv_f - mv_0$

Newton's Cradle Lab

3. Lay the ruler perfectly horizontal and put the marbles in the center touching each other.
4. From one end, roll a marble so that it hits the other four. What happens?
5. From one end, roll two marbles so that it hits the other three. What happens?
6. From one end, roll three marbles so that it hits the other two. What happens?
7. From one end, roll four marbles so that it hits the other one. What happens?
8. Roll one marble extra fast to try to get two marbles to come out at half the speed.
9. If a marble of mass m comes in at velocity v and stops and an identical marble flies out the other side, what will its velocity have to be to conserve momentum?
10. Show that momentum was conserved in steps 3-7.
11. Show that momentum would be conserved in step 7, but kinetic energy would not be conserved if two marbles came out at half the speed.

4. One flies out
5. Two fly out
6. Three fly out
7. Four fly out
8. Can't
9. *v*
10. *mv* is the same before and after
11. *mv* is the same before and after, but $KE = \frac{1}{2}mv^2$ will not be the same because of the v^2

$$KE = KE$$
$$\frac{1}{2}m(2v)^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$
$$2mv^2 \neq mv^2$$

This video is just FYI about semi truck trailers and crashes. Note that many old trailers are still on the road.

(Most underride guards fail to stop deadly crashes - IIHS news – YouTube)

04-03 ELASTIC AND INELASTIC COLLISIONS

$F\Delta t = mv_f - mv_0$

- Kinetic Energy
  - Energy of motion
  - $KE = \frac{1}{2}mv^2$

Demo energy lost to heat by smashing two steel balls together

04-03 ELASTIC AND INELASTIC COLLISIONS

$F\Delta t = mv_f - mv_0$

- Subatomic – kinetic energy often conserved
- Macroscopic – kinetic energy usually not conserved
  - Converted into heat
  - Converted into distortion or damage

Demo energy lost to heat by smashing two steel balls together

04-03 ELASTIC AND INELASTIC COLLISIONS

$F\Delta t = mv_f - mv_0$

- Elastic – kinetic energy conserved
- Inelastic – kinetic energy not conserved
- Completely inelastic – the objects stick together

SUV crash test video
NASCAR video
Crash test humor

- Bumpers are supposed to protect your car from minor accidents and provide elastic collisions are low speeds

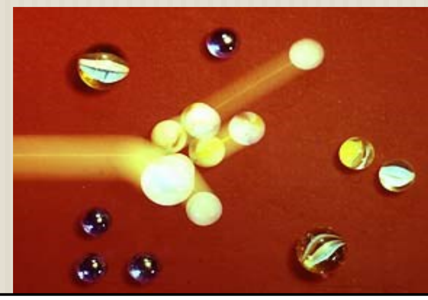
(Huge cost of mismatched bumpers by IIHS)

04-03 ELASTIC AND INELASTIC COLLISIONS

$F\Delta t = mv_f - mv_0$

- You are playing marbles. Your 0.10 kg shooter traveling at 1.0 m/s hits a stationary 0.050 kg cat's eye marble. If it is an elastic collision what are the velocities after the collision?

- v_c = 1.33 m/s
- v_s = .333 m/s



Momentum

$$m_s v_{0s} + m_c v_{0c} = m_s v_{fs} + m_c v_{fc}$$
$$(0.1\ kg)\left(1\frac{m}{s}\right) + (0.05\ kg)(0) = (0.1\ kg)v_{fs} + (0.05\ kg)v_{fc}$$
$$0.1\ kg\frac{m}{s} = (0.1\ kg)v_{fs} + (0.05\ kg)v_{fc}$$
$$v_{fs} = 1\ m/s - 0.5\ v_{fc}$$

Kinetic Energy

$$\frac{1}{2}m_s v_{0s}^2 + \frac{1}{2}m_c v_{0c}^2 = \frac{1}{2}m_s v_{fs}^2 + \frac{1}{2}m_c v_{fc}^2$$
$$\frac{1}{2}(0.1\ kg)\left(1\frac{m}{s}\right)^2 + 0 = \frac{1}{2}(0.1\ kg)v_{fs}^2 + \frac{1}{2}(0.05\ kg)v_{fc}^2$$
$$0.05\ J = (0.05\ kg)v_{fs}^2 + (0.025\ kg)v_{fc}^2$$
$$v_{fs}^2 + 0.5\ v_{fc}^2 = 1\ \left(\frac{m}{s}\right)^2$$
$$\left(1\frac{m}{s} - 0.5v_{fc}\right)^2 + 0.5\ v_{fc}^2 = 1\ \left(\frac{m}{s}\right)^2$$
$$1\ \left(\frac{m}{s}\right)^2 - \left(1\frac{m}{s}\right)v_{fc} + 0.25\ v_{fc}^2 + 0.5\ v_{fc}^2 = 1\ \left(\frac{m}{s}\right)^2$$
$$-\left(1\frac{m}{s}\right)v_{fc} + 0.75\ v_{fc}^2 = 0$$
$$v_{fc}\left(-1\frac{m}{s} + 0.75\ v_{fc}\right) = 0$$

$$v_{fc} = 0 \; or \; 1.33 \; m/s$$
$$v_{fs} = 1\frac{m}{s} - 0.5 \; v_{fc}$$
$$V_{fs} = 1 \; m/s - 0.5(1.33 \; m/s) = 0.333 \; m/s$$

04-03 ELASTIC AND INELASTIC COLLISIONS

$F\Delta t = mv_f - mv_0$

- Police will sometime reconstruct car accidents. In one accident, the cars stuck together and slid 12 m before they stopped. They measure the coefficient of friction as 0.70. The blue car's mass is 1100 kg and was sitting still at a stop sign when it was hit by the red car whose mass is 990 kg. How fast was the red car going when it hit the blue car?

Work backwards

Find acceleration after collision:

$$v^2 = v_0^2 + 2a(d - d_0)$$
$$0^2 = v_0^2 + 2a(12\,m - 0m)$$
$$-\frac{v_0^2}{24\,m} = a$$

Forces (cars stuck together m=2090 kg):

$$\mu F_N = ma$$
$$-(0.70)(mg) = ma$$
$$-(0.70)\left(9.8\frac{m}{s^2}\right) = -\frac{v_0^2}{24\,m}$$
$$164.64\frac{m^2}{s^2} = v_0^2$$
$$v_0 = 12.83\frac{m}{s}$$

Use momentum (v_0 is now final *v*):

$$m_1 v_{01} + m_2 v_{02} = m_1 v_1 + m_2 v_2$$
$$(1100\,kg)\left(0\frac{m}{s}\right) + (990\,kg)v_{02} = (1100\,kg)\left(12.83\frac{m}{s}\right) + (990\,kg)\left(12.83\frac{m}{s}\right)$$
$$(990\,kg)v_{02} = 26817.2\,kg\frac{m}{s}$$
$$v_{02} = 27.1\frac{m}{s}$$

- What can you do to stay safer in a car crash?

Keeping children safe in crashes: Overview – YouTube IIHS

Keeping children safe in crashes: Rear facing IIHS

Keeping children safe in crashes: Forward facing – YouTube IIHS

Keeping children safe in crashes: Booster seats – YouTube IIHS

Reducing Your Risks In The Crash – YouTube IIHS

04-04 ANGULAR MOMENTUM

In this lesson you will…

- Understand the analogy between angular momentum and linear momentum.
- Observe the relationship between torque and angular momentum.
- Apply the law of conservation of angular momentum.

Not in OpenStax High School Physics
OpenStax College Physics 2e 10.5

04-04 ANGULAR MOMENTUM

- Linear momentum
  - $p = mv$
- Angular momentum
  - $L = I\omega$
  - Unit:
  - kg m^2/s
  - ω must be in rad/s
- When you rotate something you exert a torque.
- More torque = faster change in angular momentum
- $\tau_{net} = \frac{\Delta L}{\Delta t}$
- Like $F = \frac{\Delta p}{\Delta t}$

Demo with the rotation rods

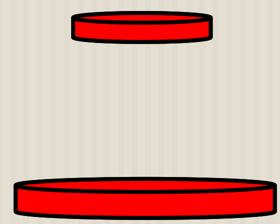
I is like *m* for rotational motion

04-04 ANGULAR MOMENTUM

- Linear momentum of a system is conserved if $F_{net} = 0$
  - $p_0 = p_f$
- Angular momentum of a system is also conserved if $\tau_{net} = 0$
  - $L_0 = L_f$

04-04 ANGULAR MOMENTUM

- A 10-kg solid disk with r = 0.40 m is spinning at 8 rad/s. A 9-kg solid disk with r = 0.30 m is dropped onto the first disk. If the second disk was initially not rotating, what is the angular velocity after the disks are together?
  - ω = 5.31 rad/s
- What was the torque applied by the first disk onto the second if the collision takes 0.01 s?
  - τ = 215 Nm



Disk 1:

$$\omega_0 = 8\frac{rad}{s}, r = 0.4\, m, m = 10\, kg$$
$$I = \frac{1}{2}MR^2 = \frac{1}{2}(10\, kg)(0.4\, m)^2 = 0.8\, kg \cdot m^2$$

Disk 2:

$$\omega_0 = 0\frac{rad}{s}, r = 0.3\, m, m = 9\, kg$$
$$I = \frac{1}{2}MR^2 = \frac{1}{2}(9\, kg)(0.3\, m)^2 = 0.405\, kg\, m^2$$
$$L = I\omega$$
$$L_0 = L_f$$
$$(0.8\, kgm^2)\left(8\frac{rad}{s}\right) + (0.405\, kgm^2)(0) = (0.8\, kgm^2 + 0.405\, kgm^2)\omega$$
$$6.4\, kgm^2 = (1.205\, kgm^2)\omega$$
$$\omega = 5.31\, rad/s$$

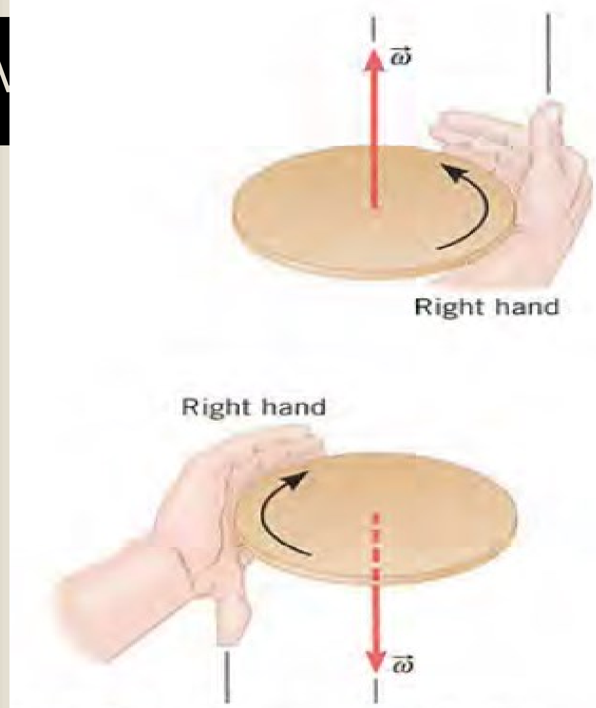
$$\tau = \frac{\Delta L}{\Delta t}$$
$$\tau = \frac{0.405\, kgm^2\left(5.31\frac{rad}{s}\right) - 0}{0.01\, s} = 215.055\, Nm$$

04-04 ANGULAR MOMENTUM

- Angular Momentum conserved if net external torque is zero
- Linear Momentum conserved if net external force is zero
- Kinetic Energy conserved if elastic collision

04-04 ANGULAR MOMENTUM

- Direction of angular quantities
  - Right-hand Rule
  - Hold hand out with thumb out along axis
  - Curl your fingers in direction of motion (you may have to turn your hand upside down)
  - vector in direction of thumb



04-04 ANGULAR MOMENTUM

- A person is holding a spinning bicycle wheel while he stands on a stationary frictionless turntable. What will happen if he suddenly flips the bicycle wheel over so that it is spinning in the opposite direction?

Consider the system of turntable, person, and wheel. The total angular momentum before is **L** upward. Afterward, the total angular momentum must be the same. If the wheel is upside down, its angular momentum is **–L** so the angular momentum of the person must be **+2L.** So the person rotates in the direction the wheel was initially spinning.